Conférence Francophone sur la Modélisation Mathématique en Biologie et en Médecine – Craiova, 12-14 juillet 2006

The analysis of the dynamics of a population in a stratified environment

Cornelia Ciutureanu

Institut de Statistique Mathématique et Mathématique Appliquée "Caius Iacob-Gheorghe Mihoc", Bucarest.

We consider the mathematical model of an aged-structured population that diffuses into a heterogeneous habitat composed of layers in which the life conditions are different.

We denote the domain by $\Omega = (0, a^+) \times (y_0, y_n)$ and the boundaries by

$$\Gamma_0 = \{(0, y) \ ; \ y \in (y_0, y_n)\}, \qquad \Gamma_{a^+} = \{(a^+, y) \ ; \ y \in (y_0, y_n)\},$$
$$\Gamma_{y_j} = \{(a, y_j) \ ; \ a \in (0, a^+)\}, \quad j = 0, \ldots, n.$$

The model is composed of the equations

$$\frac{\partial p_j}{\partial t} + \frac{\partial p_j}{\partial a} + \mu_j(a, S_j(t,y)) - K_j(a, S_j(t,y)) \frac{\partial^2 p_j}{\partial y^2} = 0, \qquad \text{in } (0, a^+) \times (0, T) \times (y_{j-1}, y_j),$$

with the initial conditions

$$p_j(a, 0, y) = p_j^0(a, y), \qquad \text{in } (0, a^+) \times (y_{j-1}, y_j), \qquad j = 1, 2, \ldots, n,$$

the conditions at the interfaces between two layers

$$\begin{aligned} p_j &= p_{j+1} && \text{on } (0,T) \times \Gamma_j, \\ K_j(a, S_j(t,y)) \frac{\partial p_j}{\partial y} &= K_{j+1}(a, S_{j+1}(t,y)) \frac{\partial p_{j+1}}{\partial y} && \text{on } \Gamma_j \times (0,T), \end{aligned}$$

for $j = 1, 2, \ldots, n-1$, and the boundary conditions

$$\begin{aligned} K_1(a, S_1(t,y)) \frac{\partial p_1}{\partial y} &= 0 \quad \text{on } (0,T) \times \Gamma_0, \\ K_n(a, S_n(t,y)) \frac{\partial p_n}{\partial y} &= 0 \quad \text{on } (0,T) \times \Gamma_n, \\ p_j(0, t, y) &= \int_0^{a^+} \beta_j(a) p_j(a, t, y) da, \end{aligned}$$

where

$$S_j(t,y) = \int_0^{a^+} \gamma_j(a) p_j(a, t, y) da.$$

We shall develop a FEMLAB algorithm for studying this problem and we shall analyze the behaviour of the solution with respect to the changes of the model parameters $(K_i, \beta_i, \gamma_i, p_i^0)$.

corneliaciutureanu@yahoo.com