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Necessary conditions of optimality for a Volterra three species system

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Consider an ecosystem consisting on a herbivorous species (the number of individuals of which is y_1), a carnivorous one (y_2), and a plant, the quantity of which is y_3. We introduce now some control variables (u, v), $0 \leq u \leq 1$, $0 \leq v \leq 1$, where $1 - u$ represents the rate of separation between the herbivorous and the carnivorous, while $1 - v$ is the rate of separation between the herbivorous and the plant. The controlled system is

$$\begin{cases} y_1' = y_1 (a_1 - b_1 y_2 u + c_1 y_3 v), \\ y_2' = y_2 (-a_2 + b_2 y_1 u), \\ y_3' = y_3 (a_3 - b_3 y_1 v), \end{cases} \quad \text{on } [0, T], \tag{1}$$

where $a_i, b_i, c_1 > 0$ are given constants, $i = 1, 2, 3$.

Suppose that the sizes of the populations at the initial moment are given by

$$y_i (0) = y_i^0 > 0, \quad i = 1, 2, 3. \tag{2}$$

We want to find the control (u, v) such that, in the end of the time interval $[0, T]$, the sum of the sizes of the three species to be maximal. This means we have to solve the optimal control problem

$$Min \{-y_1 (T) - y_2 (T) - y_3 (T)\}, \tag{3}$$

subject to $(1) - (2)$. We deduce that the optimal control is of bang-bang type and, depending on the signs of $-b_1 + b_2$ and $c_1 - b_3$, we find the number of switching points.

Next we study a similar problem in the case when the sizes of the species depend not only on the moment, but also on the position in the habitat (modelled by a bounded domain $\Omega \subset \mathbb{R}^3$). The densities of the three populations, $y_i (t, x)$, $i = 1, 2, 3$ verify on $Q = (0, T) \times \Omega$ the partial differential system of parabolic type

$$\begin{cases} \dfrac{\partial y_1}{\partial t} &= \alpha_1 \Delta y_1 + y_1 \left(a_1 - b_1 y_2 u + c_1 y_3 v \right) \\ \dfrac{\partial y_2}{\partial t} &= \alpha_2 \Delta y_2 + y_2 \left(-a_2 + b_2 y_1 u \right) \\ \dfrac{\partial y_3}{\partial t} &= \alpha_3 \Delta y_3 + y_3 \left(a_3 - b_3 y_1 v \right). \end{cases} \tag{4}$$

One associates some Neumann boundary conditions and nonnegative initial conditions.

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