Links arising from braid monodromy factorizations (BMF)

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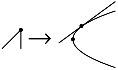
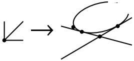
Joint with Meirav Amram and Mina Teicher

Braid monodromy studies the topology of curve configurations on 2-dimensional complex surfaces (or branch curves) and also of plane curves. These computations fit into a program started by Moishezon-Teicher using BMF as an invariant of connected components of the moduli space of surfaces of general type.

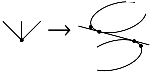
Degenerations and regenerations of surfaces make calculations more manageable for the rich structure studied using monodromy and related groups.



Partial regeneration in a nbhd of a 2-pt.

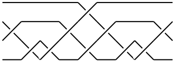


Partial reg in a nbhd of the 3-pt of type I.



Partial reg in a nbhd of the 3-point type II.

We take the BMF of these local configurations, closing the related braids to obtain links.



The braid for the complete reg for a 2-pt.

Local Propositions

1. The indices i and j of the two components of the curve that meet at a singularity are the indices of the strands in the resulting braid for that singularity.

2. The locally contributed skeletons stay intact regardless of any conjugations coming from singularities appearing above it on the braid monodromy table.

Let S be an arrangement in $\mathbb{CP}^2$ and $r(S)$ the 180° rotation of S around a horizontal line.

Global Rotation Theorem

The local contributions to the BMF obtained from S and $r(S)$ are related by a 180° rotation around a line parallel to the braid.

Moreover the closures of these braids give the same links.

The expected result is not always the case:

Theorem: 2-points and 3-points of type I

The closure of the braid for the complete regeneration for a k-point is a k-component link where each component is itself unknotted and the each pair of the k components has linking number four.

This is the torus link $T(k,k)$ with each component replaced by its $(2,1)$-cable.

Theorem: 3-points of type II

The closure of the braid for the complete regeneration for a 3-point of the second type is a four-component link where each component is itself unknotted, two of which have linking number one with two others that have linking number four that each have linking number two with each of the first two.

This is the torus link $T(3,3)$ with two strands (associated with the conics) replaced by their $(2,1)$-cables and one strand (associated with the line) replaced by its $(2,2)$-cable.

Rerences include:

Boris Moishezon and Mina Teicher,
Braid group technique in complex geometry I, II, III, IV, V.